GUIDED WAVES IN A LEFT-HANDED MATERIAL GUIDING FILM WITH A FERRITE CLADDING

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Manuscript No.: Instasci01101022. Available online at: www.instasci.info. ©2011Instasci Journals

Abstract: We investigate TE guided modes at microwave frequencies in an asymmetric waveguide structure with a left-handed material guiding layer surrounded by a linear dielectric substrate and a gyromagnetic ferrite cladding. The effect of the gyromagnetic ferrite layer parameters on the dispersion properties of the waveguide structure is investigated in details. It is found that the effective wave index of the structure is negative as if the overall structure is a left-handed material. A considerable effect of the gyromagnetic ferrite layer on the dispersion properties of the structure is observed. The power propagating in each layer is also studied.

Keywords: Dispersion relation, gyromagnetic ferrite, Left-handed materials, waveguides.

INTRODUCTION

In the past few years many scientists and engineers have been working on a novel material called left-handed material (LHM) or metamaterial due to its unusual electromagnetic features. This kind of materials was first theoretically studied by Veselago 1. He assumed a hypothetical medium of simultaneously negative electric permittivity $\varepsilon$ and magnetic permeability $\mu$ and predicted a number of unusual properties of electromagnetic waves propagating in such a medium. Among these properties was the reversal of Doppler effect and the negative index of refraction. Recently, with the realization of microwave and optical devices using LHMs, new structures including waveguides containing LHMs have been in deep concern, which may have prominent applications in the future 2. The peculiar properties of LHMs open up a wide range of applications including invisibility cloaking 3, sub-diffraction imaging 4, microstrip patch antenna 5, and optical waveguide sensing 6.

Boardman et al. studied both linear and nonlinear surface waves localized at an interface separating a left-handed and a right-handed materials 7. They demonstrated that the interface can support both TE and TM polarized surface waves.
The symmetric waveguide structure with LHM guiding layer has been analyzed and some properties of the guiding modes have been investigated. Moreover, the asymmetric waveguide structure with LHM guiding layer has also been studied. The dispersion properties of a slab waveguide with an anisotropic LHM core whose permittivity tensor is partially negative have been studied in detail. In 2006, Sabah et al. presented the reflected and transmitted powers due to the interaction of electromagnetic waves with a LHM. An asymmetric three-layer slab waveguide with a LHM layer surrounded by metal and air has been investigated. Research has been focused mostly on surface waves and resonances formed at metal-dielectric interfaces called surface plasmons and more recently on photonic crystals.

Recently there has been considerable interest of magneto static waves because of their applications in microwave integrated circuits and signal processing such as band pass filters and resonator filters. Most of these studies were carried out in waveguide structures with right handed materials. Few studies have concerned with waveguide structures including LHMs and ferrite materials.

In this work, we study guided electromagnetic waves supported by a three-layer structure consisting of a LHM film surrounded by a ferrite cover and a linear dielectric substrate. Dispersion relationships and numerical results for the effective wave number of these modes as a function of the frequency will be obtained and discussed.

**MATHEMATICAL ANALYSIS**

**Dispersion relation**

A schematic diagram of the waveguide structure under consideration is shown in Fig.1. A static biasing magnetic field $H_0$ is applied in the $+y$ direction. We assume a guiding layer made of a LHM occupies the region $0 < z < d$ bounded by a semi-infinite ferrite (YIG) cover filling the space $z > d$ and a semi-infinite linear dielectric substrate in the region $z < 0$. The guiding LHM layer is assumed to be dispersive with the permittivity and the permeability obey the Drude model describing frequency dependence of the structured LHM. This model shows that the structured LHM have a range of frequencies over which the index of refraction is negative. In such a model $\varepsilon_2$ and $\mu_2$ of the LHM take the form

$$\varepsilon_2 = 1 - \frac{\omega_p^2}{\omega^2 + i \gamma \omega}, \quad \mu_2 = 1 - \frac{F \omega_p^2}{\omega^2 - \omega_0^2 + i \Gamma \omega}$$  \hspace{1cm} (1)
where $\omega$ is the frequency of the incident light, $\omega_p$ and $\omega_0$ are the electronic and magnetic plasma frequencies, $\gamma$ and $\Gamma$ are the damping rates relating to the absorption of the material, and $F$ is the fractional area of the unit cell occupied by the split ring.

\[ \mu(\omega) = \begin{pmatrix} \mu_{xx} & 0 & \mu_{xz} \\ 0 & \mu_B & 0 \\ -\mu_{xz} & 0 & -\mu_{xx} \end{pmatrix}, \]  
(2)

where

\[ \mu_{xx} = \mu_B \left( \frac{\omega_{0, f}}{\omega_{0, f}^2 - \omega^2} + \frac{\omega_m}{\omega_m^2 - \omega^2} \right), \quad \mu_{xx} = i\mu_B \frac{\omega_0\omega_{m}}{\omega_{0, f}^2 - \omega^2}, \]  
(3)

and $\mu_B = \mu_{xx} + \frac{\mu_{xx}^2}{\mu_{xx}}$ with $\mu_B$ is the usual Polder tensor element, $\omega_{0, f} = \gamma_f \mu_0 H_0$, $\omega_m = \gamma_f \mu_0 M_0$ is the DC applied magnetic field, $\gamma_f$ is the gyro magnetic ratio, $M_0$ is the DC magnetization of the magnetic insulator.

Fig. 1. Schematic diagram of a waveguide structure comprising a left-handed material guiding layer and a ferrite cladding.

The two claddings marked with region 1 (substrate) and region 3 (cladding) have parameters $(\varepsilon_1$ and $\mu_1)$ and $(\varepsilon_3$ and $\mu_3)$, respectively. The cladding is assumed to be a gyromagnetic ferrite layer which is described by a magnetic permeability tensor as $^{13-15}$.

Only TE modes will be considered which are assumed to propagate along $x$-axis with wave number $k_0 \beta$ and angular frequency $\omega$. The electric and magnetic fields, in this case, are given by
Upon solving Maxwell’s equations in the three layers, the eclectic field can be written as

\[ E_y^{(1)} = A e^{i(kz)} \]  
\[ E_y^{(2)} = B \cos(k_2z) + C \sin(k_2z) \]  
\[ E_y^{(3)} = D e^{-k_3(z-d)} \]

where \( k_1 = k_0 \sqrt{\beta^2 - \varepsilon_s \mu_s} \), \( k_2 = k_0 \sqrt{\varepsilon_s \mu_s - \beta^2} \), \( k_3 = k_0 \sqrt{\beta^2 - \varepsilon_s \mu_s} \), \( A \), \( B \), \( C \), and \( D \) are the wave amplitude coefficients determined from the boundary conditions.

The nonzero magnetic field components in the cover \((z > d)\) are given by:

\[ H_z^{(3)} = \left( \frac{\mu_{ss} k_3 + i k_{ss} \mu_{ss}}{i \omega \mu_0 \mu_{ss} \mu_s} \right) E_y^{(3)} \]
\[ H_z^{(3)} = \left( \frac{\mu_{ss} k_3 - i k_{ss} \mu_{ss}}{i \omega \mu_0 \mu_{ss} \mu_s} \right) E_y^{(3)} \]

while in the substrate layer

\[ H_z^{(1)} = \frac{ik_1}{\omega \mu_1} A e^{k_1z} \], \[ H_z^{(1)} = \frac{k_1}{\omega \mu_1} A e^{k_1z} \]

In region \(0 < z < d\), the magnetic field components are given by

\[ H_z^{(2)} = \frac{-ik_2}{\omega \mu_2} [B \sin(k_2z) - C \cos(k_2z)] \]
\[ H_z^{(2)} = \frac{k_2}{\omega \mu_2} [B \cos(k_2z) + C \sin(k_2z)] \]

Matching the field component \( H_x \) and \( E_y \) at the boundaries \( z = 0 \) and \( z = d \), the following dispersion relation is obtained
\[ \tan(k_zd) = \frac{k_1\mu_0\mu_t + \mu_{sz}k_3 - i k\mu_{sz}}{k_2\mu_0\mu_t - \mu_{sz}k_3 - k_3\mu_t} \]  
\[ \frac{k_1\mu_0\mu_t + \mu_{sz}k_3 - i k\mu_{sz}}{k_2\mu_0\mu_t - \mu_{sz}k_3 - k_3\mu_t} \]  

B. Power Flow

It is instructive to study the time-average power transported by each layer. We have

\[ P = \frac{1}{2}\int (E \times H^*)dz \]  

Therefore, the time-average power flowing in the substrate, the film, and the cladding layers are respectively given by

\[ P_1 = \frac{ikA^2}{2k_1\omega\mu_1} \]  
\[ P_2 = \frac{ik}{\omega\mu} \left[ \frac{k_zd + \sin(2k_zd)}{4k_2} + \frac{k_z^2\mu_t^2}{2k_z\mu_t^2} - \frac{k_z^2\mu_t^2\sin(2k_zd)}{4k_2\mu_t^2k_2} \right] \]  
\[ P_3 = \frac{\mu_{sz}k_3 + ik\mu_{sz}}{i\omega\mu_0\mu_{sz}\mu_t} \left[ A(\cos(k_zd) + \frac{k_1\mu_0}{k_z\mu_t}A\sin(k_zd)) \right]^2 \]  

**Numerical Results and Discussion**

In order to study the dispersion characteristics of the proposed structure, the dispersion relation has to be solved numerically to find the effective wave index \( \beta \) as a function of the angular frequency \( \omega \). The frequency range should be chosen carefully to guarantee that \( \mu_2(\omega) \) and \( \varepsilon_2(\omega) \) are negative. The numerical calculations were carried out using the following parameters for the ferrite (YIG) cladding \( \varepsilon_3 = 1, \gamma = 1.76 \times 10^{11} \text{ s}^{-1} \text{T}^{-1}, \mu_0 M = 0.175 \text{ T}, \) and \( \omega_m = \gamma \mu_0 H_0 \) and the following parameters for the LHM guiding layer \( \omega_p = 10.0 \text{ GHz}, \omega_0 = 4.0 \text{ GHz}, \gamma = 0.03\omega_p, \Gamma = 0.03\omega_0, F = 0.56, \) and \( d = 120 \mu\text{m}. \) The substrate is assumed to be a lossless dielectric with \( \varepsilon_1 = 2.45, \mu_1 = 1. \)

In Figure 2, the effective wave index, \( \beta \), is plotted versus the frequency for different values of \( \mu_0 H_0 \). It is noticed that the effective wave index is smoothly increasing with frequency. The figure also reveals that for a given frequency, \( \beta \) can be enhanced by reducing the applied external magnetic field. The range of frequencies over which the structure can support guided waves is strongly dependent on the static biasing magnetic field \( H_0 \) as can be seen from Fig. 2. This frequency range can be considerably enhanced by increasing the value of \( \mu_0 H_0 \). For \( \mu_0 H_0 = 0.18, 0.19, 0.20, \) and \( 0.21 \) the structure can support guided waves in the frequency ranges \( 4.1<\omega<4.8, 4.1<\omega<5.1, 4.1<\omega<5.4, \)
and $4.1<\omega<5.7$, respectively. An important feature can be observed from the figure: the effective wave index of the structure is negative as if the overall structure is left-handed material. This means the reversal of the energy flow and that the group velocity and the phase velocity are in opposite directions. In Fig. 3, the effect of the DC magnetization of the magnetic insulator on the dispersion characteristics is studied. As can be seen, the dispersion curves shift towards higher frequencies with increasing $\mu_0 M$. Figure 4 shows the effect of the gyromagnetic ratio $\gamma_l$ on the dispersion curves. The DC magnetization of the magnetic insulator and the gyromagnetic ratio almost have the same effect on the dispersion characteristics of the proposed structure. The dispersion curves shift towards higher frequencies with increasing either $\mu_0 M$ or $\gamma_l$. The computed dispersion curves of TE-polarized guided waves for the forward direction for different values of the film thickness is shown in Fig. 5. The gyromagnetic ferrite layer parameters ($H_0$, $\mu_0 M$, $\gamma_l$) have a considerable effect on the dispersion properties of the structure compared with the thickness of the film. A slight change in the effective wave index is observed with increasing the guiding layer thickness.

**Fig. 2.** Dispersion curves of TE-polarized guided waves for different values of $\mu_0 H_0$ for $\varepsilon_3 = 2.45$, $\mu_1 = 1$, $\omega_p = 10.0$ GHz, $\omega_0 = 4.0$ GHz, $\gamma = 0.03\omega_p$, $\Gamma = 0.03\omega_0$, $F = 0.56$, $d = 120$ $\mu$m, $\varepsilon_3 = 1$, $\gamma_l = 1.76\times10^{11}$ $s^{-1}$, $\mu_0 M = 0.175$ T, and $\omega_0 f = \gamma_l \mu_0 H_0$. 

Fig. 3. Dispersion curves of TE-polarized guided waves for different values of $\mu_0 M$ for $\varepsilon_1 = 2.45$, $\mu_1 = 1$, $\omega_p = 10.0 \text{ GHz}$, $\omega_0 = 4.0 \text{ GHz}$, $\gamma = 0.03\omega_p$, $\Gamma = 0.03\omega_0$, $F = 0.56$, $d = 120 \mu\text{m}$, $\varepsilon_3 = 1$, $\gamma_f = 1.76 \times 10^{11} \text{s}^{-1}\text{T}^{-1}$, $\mu_0 H_0 = 0.21$, and $\omega_{0f} = \gamma_f \mu_0 H_0$.

Fig. 4. Dispersion curves of TE-polarized guided waves for different values of $\gamma$ for $\varepsilon_1 = 2.45$, $\mu_1 = 1$, $\omega_p = 10.0 \text{ GHz}$, $\omega_0 = 4.0 \text{ GHz}$, $\gamma = 0.03\omega_p$, $\Gamma = 0.03\omega_0$, $F = 0.56$, $d = 120 \mu\text{m}$, $\varepsilon_3 = 1$, $\mu_0 M = 0.175$, $\mu_0 H_0 = 0.21$, and $\omega_{0f} = \gamma_f \mu_0 H_0$. 

Fig. 5. Dispersion curves of TE-polarized guided waves for different values of $d$ for $\varepsilon_1 = 2.45$, $\mu_1 = 1$, $\omega_p = 10.0$ GHz, $\omega_0 = 4.0$ GHz, $\gamma = 0.03\omega_p$, $\Gamma = 0.03\omega_0$, $F = 0.56$, $d = 120$ $\mu$m, $\varepsilon_3 = 1$, $\gamma_f = 1.76 \times 10^{11}$ s$^{-1}$, $\mu_0 M = 0.175$, $\mu_0 H_0 = 0.1$, and $\omega_{0f} = \gamma_f \mu_0 H_0$.

The part of the power flowing in each layer as a function of the propagating wave frequency for different values of the fractional area of the unit cell occupied by the split ring is shown in Figs. 6, 7 and 8. A number of interesting features can be seen in these figures. First, the ferrite (YIG) cladding has the highest fractional of total power. This means that the proposed structure is a strong candidate for a non-communication application of slab waveguides called optical sensing. The part of total power flowing in the cladding layer is a significant parameter in the field of slab waveguide optical sensing. The enhancement of this part enhances the sensitivity of the effective wave index to changes in the refractive index of the cladding. Second, increasing the fractional area of the unit cell enhances the power flowing in the substrate and reduces the absolute value of the powers following in the LHM guiding layer but it does not has a considerable effect on the power flowing in the ferrite cladding. Increasing $F$ reduces very slightly the part of the power flowing in the ferrite cladding. The inset in Fig. 8 is added to show this slight effect. Third, the power $P_2$ flowing in the LHM guiding layer is negative which is an important feature that can be seen in these figures. This is one of the main differences between left-handed and right-handed materials. In RHM, the Poynting's vector $S$ always forms a right-handed set with the vectors $E$ and $H$. Accordingly, for RHMs $S$ and the propagation vector $k$ are in the same direction. However, this is not the case of LHM in which $S$ and $k$ are in opposite directions. It is well known that the phase velocity and the propagation vector $k$ are in the same direction for normal materials. Thus, it is clear that LHM are substances with a so-called negative group velocity, which occurs in particular in anisotropic substances or when there is spatial dispersion. Fig. 8 emphasizes the fact that in LHM the phase velocity is opposite to the energy flow.
Finally, the dependence of the power on the frequency is obvious from Figs. 6-8. We have adopted the frequency range $4.5<\omega<5.2$. As can be seen from the figures, at low frequencies most of the power is devoted to the cladding and substrate whereas at high frequencies most of it is confined to the LHM guiding layer.

Fig. 6. The power flowing through the substrate layer versus the angular frequency for different values of $F$ for $\varepsilon_1 = 2.45$, $\mu_1 = 1$, $\omega_p = 10.0$ GHz, $\omega_0 = 4.0$ GHz, $\gamma = 0.03\omega_p$, $\Gamma = 0.03\omega_0$, $F = 0.56$, $d = 120$ $\mu$m, $\varepsilon_3 = 1$, $\gamma_f = 1.76\times10^{11}$ s$^{-1}$T$^{-1}$, $\mu_0M = 0.175$ T, $\mu_0H_0 = 0.21$, and $\omega_{0f} = \gamma_f \mu_0H_0$.

Fig. 7. The power flowing through the left-handed material guiding layer versus the angular frequency for different values of $F$ for $\varepsilon_1 = 2.45$, $\mu_1 = 1$, $\omega_p = 10.0$ GHz, $\omega_0 = 4.0$ GHz, $\gamma = 0.03\omega_p$, $\Gamma = 0.03\omega_0$, $F = 0.56$, $d = 120$ $\mu$m, $\varepsilon_3 = 1$, $\gamma_f = 1.76\times10^{11}$ s$^{-1}$T$^{-1}$, $\mu_0M = 0.175$ T, $\mu_0H_0 = 0.21$, and $\omega_{0f} = \gamma_f \mu_0H_0$. 
Fig. 8. The power flowing through the cladding layer versus the angular frequency for different values of \( F \) for \( \varepsilon_2 = 2.45, \mu_1 = 1, \omega_p = 10.0 \text{ GHz}, \omega_0 = 4.0 \text{ GHz}, \gamma = 0.03\omega_p, \Gamma = 0.03\omega_0, \) \( F = 0.56, d = 120 \text{ \mu m}, \varepsilon_3 = 1, \gamma_l = 1.76\times10^{11} \text{ s}^3\text{T}^{-1}, \mu_0 M = 0.175 \text{ T}, \mu_0 H_0 = 0.21, \) and \( \omega_{0f} = \gamma_l \mu_0 H_0. \)

**CONCLUSION**

We have studied analytically the TE guided waves in a slab waveguide structure comprising a left handed material film between ferrite cover and dielectric substrate. The dispersion relation for TE guided waves has been derived and numerically investigated. It is found that the effective wave index is negative which means that the structure exhibit a LHM behavior. The range of frequencies over which the structure can support guided waves is strongly dependent on the gyromagnetic ferrite layer parameters. We noticed that the power flow are inversely proportional to the angular frequency in lossless substrate and the Ferrite cladding while it is growing with the frequency in the LHM material.

**REFERENCES**

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